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# Description Logics with Aggregates and Concrete Domains

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## Abstract

We show that extending description logics by simple aggregation functions as available in database systems may lead to undecidability of inference problems such as satisfiability and subsumption.

## 1 Motivation

Aggregation is a very useful mechanism available in many expressive representation formalisms such as database schema and query languages. Most systems provide a fixed set of aggregation functions like `sum`, `min`, `max`, `average`, `count`, which can be used over a given built-in domain, like the integers or the reals. In this paper, the generic Description Logic  $\mathcal{ALC}(\mathcal{D})$  as introduced in [Baader&Hanschke,1991] is extended by aggregation.  $\mathcal{ALC}(\mathcal{D})$  is an extension of the well-known description language  $\mathcal{ALC}$  (see [Schmidt-Schauß&Smolka,1991; Hollunder *et al.*,1990; Donini *et al.*,1991]) by so-called *concrete domains*. In the basic language,  $\mathcal{ALC}$ , concepts can be built using propositional operators, (i.e., *and* ( $\sqcap$ ), *or* ( $\sqcup$ ), and *not* ( $\neg$ )), and value restrictions on those individuals associated to an individual via a certain role. These include *existential* restrictions like in ( $\exists$ has\_child.Girl) as well as *universal* restrictions like ( $\forall$ has\_child.Human). Additionally, in  $\mathcal{ALC}(\mathcal{D})$ , abstract individuals which are described using  $\mathcal{ALC}$  can now be related to values in a *concrete domain* (e.g., the integers or strings) via *features*, i.e., functional roles. This allows us to describe managers that spend more money than they earn by

Manager  $\sqcap$  (less(income, expenses)).

In our extension of  $\mathcal{ALC}(\mathcal{D})$ , aggregation is viewed as a means to define new features. In Figure 1, a person, *Josie*, is given who spends, in some months, more money than she earns, and in others less. If we want to know the difference between income and expenses for a whole year, we have to consider the sum over all months. Then we can state that or ask whether *Josie* is an instance of

Human  $\sqcap$  ( $\exists$ year.less( $\text{sum}(\text{month} \circ \text{income})$ ,  
 $\text{sum}(\text{month} \circ \text{expenses}))$ ),

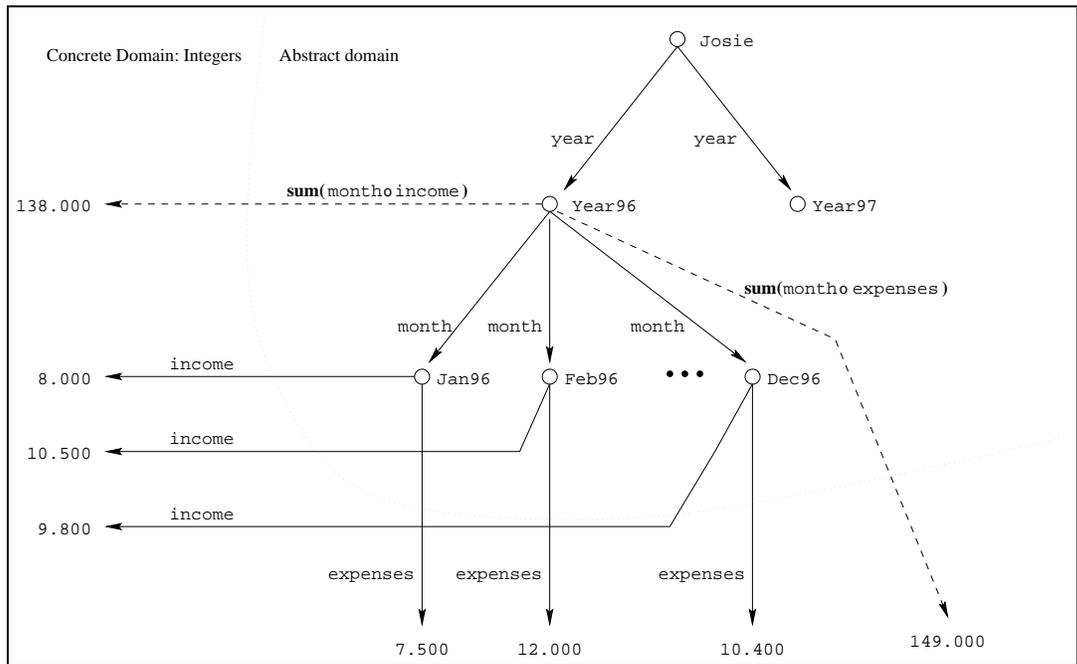


Figure 1: Example for aggregation

where the complex feature  $\text{sum}(\text{month} \circ \text{income})$  relates an individual to the sum over all values reachable over  $\text{month}$  followed by  $\text{income}$ . This new, complex feature is built using the aggregation function  $\text{sum}$ , the role name  $\text{month}$ , and the feature  $\text{income}$ .

In this paper, we present a generic extension of  $\mathcal{ALC}(\mathcal{D})$  by aggregation that is based on this idea of introducing new “aggregated features”. Unfortunately, it turns out that, given a concrete domain together with aggregation functions satisfying some very weak conditions, this extension has an undecidable satisfiability problem. Moreover, this result can even be tightened: extending  $\mathcal{FL}_0$ , a very weak Description Logic allowing for conjunction and universal value restrictions only, by a weak form of aggregation already leads to undecidability of satisfiability and subsumption.

For database research, these results are, for example, of interest in the context of intensional reasoning in the presence of aggregation, as considered in [Ross *et al.*,1998; Gupta *et al.*,1995; Mumick & Shmueli,1995; Levy & Mumick,1996; Srivastava *et al.*,1996]. They are not comparable with the undecidability results presented in [Mumick & Shmueli,1995] since our prerequisites are weaker and no recursion mechanisms are used. Neither are they contained in the undecidability results in [Ross *et al.*,1998]: the results presented there concern constraints involving multiplication and addition as well as rather complex aggregation functions like  $\text{average}$  or  $\text{count}$ —in contrast to the results presented here.

## 2 Preliminaries: The basic Description Logic $\mathcal{ALC}(\mathcal{D})$

In this section,  $\mathcal{ALC}(\mathcal{D})$ , the Description Logic underlying this investigation, is presented.  $\mathcal{ALC}(\mathcal{D})$  is an extension of the well-known Description Logic  $\mathcal{ALC}$  (see [Schmidt-Schauß & Smolka,1991; Hollunder *et al.*,1990; Donini *et al.*,1991; 1995]) by so-called *concrete domains*. First, we formally specify a concrete domain.

### Definition 1 (Concrete Domains)

A *concrete domain*  $\mathcal{D} = (\text{dom}(\mathcal{D}), \text{pred}(\mathcal{D}))$  consists of

- a set  $\text{dom}(\mathcal{D})$  (the domain), and
- a set of predicate symbols  $\text{pred}(\mathcal{D})$ .

Each predicate symbol  $P \in \text{pred}(\mathcal{D})$  is associated with an arity  $n$  and an  $n$ -ary relation  $P^{\mathcal{D}} \subseteq \text{dom}(\mathcal{D})^n$ .

In [Baader & Hanschke,1991], concrete domains are restricted to so-called *admissible* concrete domains in order to keep the inference problems of this extension decidable. We recall that, roughly spoken, a concrete domain  $\mathcal{D}$  is called *admissible* iff (1)  $\text{pred}(\mathcal{D})$  is closed under negation and contains a unary predicate name  $\top$  for  $\text{dom}(\mathcal{D})$ , and (2) satisfiability of finite conjunctions over  $\text{pred}(\mathcal{D})$  is decidable.

The syntax of  $\mathcal{ALC}(\mathcal{D})$ -concepts is defined as follows (see [Baader & Hanschke,1991]):

**Definition 2** Let  $N_C$ ,  $N_R$ , and  $N_F$  be disjoint sets of *concept*, *role*, and *feature names*. The set of  $\mathcal{ALC}(\mathcal{D})$ -concepts is the smallest set such that

1. every concept name is a concept and
2. if  $C, D$  are concepts,  $R$  is a role or a feature name,  $P \in \text{pred}(\mathcal{D})$  is an  $n$ -ary predicate name, and  $u_1, \dots, u_n$  are feature chains,<sup>1</sup> then  $(C \sqcap D)$ ,  $(C \sqcup D)$ ,  $(\neg C)$ ,  $(\forall R.C)$ ,  $(\exists R.C)$ , and  $P(u_1, \dots, u_n)$  are concepts.

In order to fix the exact meaning of these concepts, their semantics is defined in the usual model-theoretic way.

**Definition 3** An *interpretation*  $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$  consists of a set  $\Delta^{\mathcal{I}}$  disjoint from  $\text{dom}(\mathcal{D})$ , called the *domain* of  $\mathcal{I}$ , and a function  $\cdot^{\mathcal{I}}$  which maps every concept to a subset of  $\Delta^{\mathcal{I}}$ ,

<sup>1</sup>A feature chain  $u = f_1 \circ \dots \circ f_m$  is a sequence of features  $f_i$ .

every role to a subset of  $\Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ , and every feature name  $f \in N_F$  to a partial function  $f^{\mathcal{I}} : \Delta^{\mathcal{I}} \rightarrow \Delta^{\mathcal{I}} \cup \text{dom}(\mathcal{D})$ . Furthermore,  $\mathcal{I}$  has to satisfy the following properties

$$\begin{aligned}
 (C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
 (C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
 \neg C^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
 (\exists R.C)^{\mathcal{I}} &= \{d \in \Delta^{\mathcal{I}} \mid \text{Exists } e \in \Delta^{\mathcal{I}} \text{ with } (d, e) \in R^{\mathcal{I}} \text{ and } e \in C^{\mathcal{I}}\} \\
 (\forall R.C)^{\mathcal{I}} &= \{d \in \Delta^{\mathcal{I}} \mid \text{For all } e \in \Delta^{\mathcal{I}}, \text{ if } (d, e) \in R^{\mathcal{I}}, \text{ then } e \in C^{\mathcal{I}}\} \\
 P(u_1, \dots, u_n)^{\mathcal{I}} &= \{x \in \Delta^{\mathcal{I}} \mid (u_1^{\mathcal{I}}(x), \dots, u_n^{\mathcal{I}}(x)) \in P^{\mathcal{D}}\}
 \end{aligned}$$

where  $(f_1 \circ \dots \circ f_m)^{\mathcal{I}}(x) = f_m^{\mathcal{I}}(f_{m-1}^{\mathcal{I}}(\dots(f_1^{\mathcal{I}}(x))\dots))$ . A concept  $C$  is called *satisfiable* iff there is some interpretation  $\mathcal{I}$  such that  $C^{\mathcal{I}} \neq \emptyset$ . Such an interpretation is called a *model* of  $C$ . A concept  $D$  *subsumes* a concept  $C$  (written  $C \sqsubseteq D$ ) iff  $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$  holds for each interpretation  $\mathcal{I}$ . Two concepts are said to be equivalent (written  $C \equiv D$ ) if they mutually subsume each other. For an interpretation  $\mathcal{I}$ , an individual  $x \in \Delta^{\mathcal{I}}$  is called an *instance* of a concept  $C$  iff  $x \in C^{\mathcal{I}}$ .

As a consequence of this definition, an instance of a concept  $P(u_1, \dots, u_n)$  has necessarily an  $u_i$ -successor in  $\text{dom}(\mathcal{D})$  for each  $1 \leq i \leq n$ . Furthermore, if  $x \in \top(f)^{\mathcal{I}}$ , then  $f^{\mathcal{I}}(x) \in \text{dom}(\mathcal{D})$ . To express that an individual has no  $f$ -successor at all, we will use the abbreviation  $\text{no}_f = \forall f.(A \sqcap \neg A)$ . As  $\mathcal{ALC}(\mathcal{D})$  allows for negation and conjunction of concepts, all boolean operators can be expressed, and we will use  $C \Rightarrow D$  as a shorthand for  $\neg C \sqcup D$ . Another consequence of the presence of these two operators is that subsumption and (un)satisfiability can be reduced to each other:

- $C \sqsubseteq D$  iff  $C \sqcap \neg D$  is unsatisfiable,
- $C$  is unsatisfiable iff  $C \sqsubseteq A \sqcap \neg A$  (for some concept name  $A$ ).

From the results presented in [Baader & Hanschke,1991] it follows immediately that subsumption and satisfiability are decidable for  $\mathcal{ALC}(\mathcal{D})$  concepts—given that  $\mathcal{D}$  is admissible. The authors present a tableau-based procedure that decides these and other inference problems.

### 3 Extension of $\mathcal{ALC}(\mathcal{D})$ by aggregation

In order to define aggregation appropriately, first, we will introduce the notion of *multisets*: In contrast to simple sets, in a multiset an individual can occur more than once; for example, the multiset  $\{1\}$  is different from the multiset  $\{1, 1\}$ . Multisets are needed to ensure, e.g., that *Josie*'s income is calculated correctly in the case she earns the same amount of money in more than one month.

**Definition 4 (Multisets)** Let  $S$  be a set. A *multiset*  $M$  over  $S$  is a mapping  $M : S \rightarrow \mathbb{N}$ , where  $M(s)$  denotes the number of occurrences of  $s$  in  $M$ . The set of all multisets of  $S$  is denoted  $\mathbf{MS}(S)$ .

A multiset  $M$  is said to be finite iff  $\{s \mid M(s) \neq 0\}$  is a finite set.

As the aggregation functions strongly depend on the specific concrete domains, the notion of a *concrete domain* is extended accordingly. Furthermore, the notion of *concrete features* is introduced. Those are (possibly complex) features which can be built using aggregation over roles followed by features. Then  $\mathcal{ALC}(\mathcal{D} + \Sigma)$ -concepts are defined.

**Definition 5** The notion of a concrete domain  $\mathcal{D}$  as introduced in Definition 1 is extended by a set of aggregation functions  $\mathbf{agg}(\mathcal{D})$ , where each  $\Sigma \in \mathbf{agg}(\mathcal{D})$  is associated with a partial function  $\Sigma^{\mathcal{D}}$  from the set of multisets of  $\mathbf{dom}(\mathcal{D})$  into  $\mathbf{dom}(\mathcal{D})$ .

The set of *concrete features* is inductively defined as follows:

- Each feature name  $f \in N_F$  is a concrete feature,
- chains of concrete features are concrete features,
- if  $R \in N_R$  is a role,  $f$  is a concrete feature,  $\Sigma \in \mathbf{agg}(\mathcal{D})$  is an aggregation function, then  $\Sigma(R \circ f)$  is a concrete feature.

Finally,  $\mathcal{ALC}(\mathcal{D} + \Sigma)$ -concepts are obtained from  $\mathcal{ALC}(\mathcal{D})$ -concepts by allowing, additionally, the use of concrete features  $f_i$  in a predicate restrictions  $P(f_1, \dots, f_n)$  (recall that in  $\mathcal{ALC}(\mathcal{D})$  only feature chains were allowed).

It remains to extend the semantics of  $\mathcal{ALC}(\mathcal{D})$  to the new feature forming operator:

**Definition 6 (Extended Semantics)** An  $\mathcal{ALC}(\mathcal{D} + \Sigma)$ -interpretation  $\mathcal{I}$  is an  $\mathcal{ALC}(\mathcal{D})$ -interpretation which additionally satisfies

$$(\Sigma(R \circ f))^{\mathcal{I}} = \{(x, y) \in \Delta^{\mathcal{I}} \times \mathbf{dom}(\mathcal{D}) \mid \Sigma^{\mathcal{D}}(M_x^{R \circ f}) = y\}$$

where, for  $x \in \Delta^{\mathcal{I}}$ , a feature  $f$ , and a role  $R$ ,  $M_x^{R \circ f}$  denotes the multiset over  $\mathbf{dom}(\mathcal{D})$  where the number of occurrences of  $z \in \mathbf{dom}(\mathcal{D})$  is determined by the number of  $R^{\mathcal{I}}$ -successors  $y$  of  $x$  with  $f^{\mathcal{I}}(y) = z$ , i.e. for  $z \in \mathbf{dom}(\mathcal{D})$  we have

$$M_x^{R \circ f}(z) = \#\{y \in \Delta^{\mathcal{I}} \mid (x, y) \in R^{\mathcal{I}} \text{ and } f^{\mathcal{I}}(y) = z\}.$$

We point out two consequences of this definition, which might not be obvious at first sight:

(a) If  $(R \circ f)^{\mathcal{I}}(x)$  contains individuals in  $\Delta^{\mathcal{I}}$ , then these individuals have no influence on  $M_x^{R \circ f}$ : it is defined in such a way that it takes only into account  $(R \circ f)^{\mathcal{I}}$ -successors of  $x$  in the concrete domain  $\text{dom}(\mathcal{D})$ .

(b) Aggregation functions are *partial* functions, hence  $(\Sigma(R \circ f))^{\mathcal{I}}(x)$  does not need to be defined. For example, the (standard) sum over an infinite set of numbers larger than 1 is undefined: If  $\text{dom}(\mathcal{D})$  is the set of reals, and if  $x$  has infinitely many  $R$ -successors in  $\mathcal{I}$  which all have an  $f$ -successor in the reals that is larger than 1, then  $(\text{sum}(R \circ f))^{\mathcal{I}}(x)$  is undefined. To enforce that an individual has  $f_i$ -successors in  $\text{dom}(\mathcal{D})$ , we can make use of predicate restrictions  $P(f_1, \dots, f_n)$ . Recall that for  $x \in \Delta^{\mathcal{I}}$  to be an instance of a concept  $P(f_1, \dots, f_n)$ , it is necessary that for each concrete feature  $f_i$  the value  $f_i^{\mathcal{I}}(x)$  is defined and in  $\text{dom}(\mathcal{D})$ .

### 3.1 A first undecidability result

The following theorem states that admissibility of a concrete domain does no longer guarantee decidability of the interesting inference problems:

**Theorem 7** For a concrete domain  $\mathcal{D}$  where

- $\text{dom}(\mathcal{D})$  includes the non-negative integers,
- $\text{pred}(\mathcal{D})$  contains a (unary) predicate  $P_{=1}$  that tests for equality with 1, and a (binary) equality  $P_{=}$ ,
- $\text{agg}(\mathcal{D})$  contains  $\text{min}$ ,  $\text{max}$ ,  $\text{sum}$ ,

satisfiability and subsumption of  $\mathcal{ALC}(\mathcal{D} + \Sigma)$ -concepts is undecidable.

**Remarks:** (a) The aggregation functions  $\text{min}$ ,  $\text{max}$ ,  $\text{sum}$  are supposed to be defined as usual, i.e., for multisets  $M$  over the reals (and thus also for multisets  $M$  over the non-negative integers) we have

$$\text{sum}(M) = \begin{cases} \sum_{y \in M} M(y) \cdot y & \text{if } M \text{ is finite} \\ \text{undefined} & \text{otherwise} \end{cases}$$

$$\text{min}(M) = \begin{cases} m & \text{if there exists } m \in M \text{ such} \\ & \text{that } n \geq m \text{ for all } n \in M \\ \text{undefined} & \text{if such an } m \text{ does not exist} \end{cases}$$

$$\text{max}(M) = \begin{cases} m & \text{if there exists } m \in M \text{ such} \\ & \text{that } n \leq m \text{ for all } n \in M \\ \text{undefined} & \text{if such an } m \text{ does not exist} \end{cases}$$

(b) At first sight, this undecidability result seems very restricted. Note, however, that it does not require that  $\text{dom}(\mathcal{D})$  is the set of non-negative integers, but that it just requires that  $\text{dom}(\mathcal{D})$  *contains* the non-negative integers. This makes the undecidability result not only more general, but also stronger: For example, computations over the reals are, in general, easier than computations over the non-negative integers, i.e., the first order theory of  $+, \cdot, \leq$  is undecidable over the non-negative integers, whereas it is decidable over the reals.

Furthermore, the aggregation functions  $\text{min}, \text{max}, \text{sum}$  are among those normally considered as built-in functions for databases (see, for example, [Gupta *et al.*,1995; Mumick & Shmueli,1995; Levy & Mumick,1996; Srivastava *et al.*,1996]). Finally, to test whether a certain value equals 1 or whether two values are equal is possible in all database systems with built-in predicates.

(c) We do not suppose that  $\mathcal{D}$  is admissible—although this precondition would not make the undecidability result less expressive. Nevertheless, in the sequel we will make use of the concept  $\top(f)$ . This is in accordance with the preconditions of Theorem 7 because  $\top(f)$  (if not available in  $\mathcal{D}$ ) can be introduced as abbreviation, e.g., for  $P_=(f, f)$ .

**Proof of Theorem 7:** The proof is by reduction of Hilbert’s 10th problem [Davis,1973] to satisfiability of concepts, i.e., for polynomials  $P, Q \in \mathbb{N}[x_1, \dots, x_m]$ , one can construct an  $\mathcal{ALC}(\mathcal{D} + \Sigma)$ -concept  $C_{P,Q}$  that is satisfiable iff the polynomial equation

$$P(x_1, \dots, x_m) = Q(x_1, \dots, x_m) \tag{1}$$

has a solution in  $\mathbb{N}^m$ . In the sequel, we write  $\mathbf{x}$  as shorthand for  $(x_1, \dots, x_m)$  and  $\mathbf{x}^{ij}$  as shorthand for the monomial  $x_1^{i_{j1}} \cdots x_m^{i_{jm}}$ .

The idea of the reduction is to represent the (sub)term structure of the polynomial  $P$  (resp.  $Q$ ) as a tree which is related to an instance of  $C_{P,Q}$  via the feature  $P$  (resp.  $Q$ ); see Figure 2. The polynomial  $P$  is supposed to be of the form

$$P(\mathbf{x}) = a_0 + a_1 \mathbf{x}^{i_1} + \dots + a_j \mathbf{x}^{i_j} + \dots + a_n \mathbf{x}^{i_n},$$

where all monomials  $\mathbf{x}^{i_j}$  are supposed to be different.

When building the reduction concept  $C_{P,Q}$ , one encounters three main problems: (a) We only know that  $\text{dom}(\mathcal{D})$  *contains*  $\mathbb{N}$ , but the solution of Equation 1 has to be in  $\mathbb{N}^m$ , and  $\mathcal{D}$  need not provide for a predicate that tests for being a non-negative integer. (b) It has to be assured that (the representation of) each variable  $x_i$  is associated with the same non-negative integer wherever it occurs in a model of  $C_{P,Q}$ . (c) The reduction asks for the representation of calculations such as addition, multiplication, and exponentiation.

These problems can be overcome as follows:

(a) is solved by making use of the concept  $E_g^R$ ,

$$E_g^R := (\forall R.(P_{=1}(f))) \sqcap P_=(\text{sum}(R \circ f), g),$$

whose instances have as  $g$ -successors the number of their  $R$ -successors. Hence their  $g$ -successor is in  $\mathbb{N}$  or undefined (if there are infinitely many  $R$ -successors).

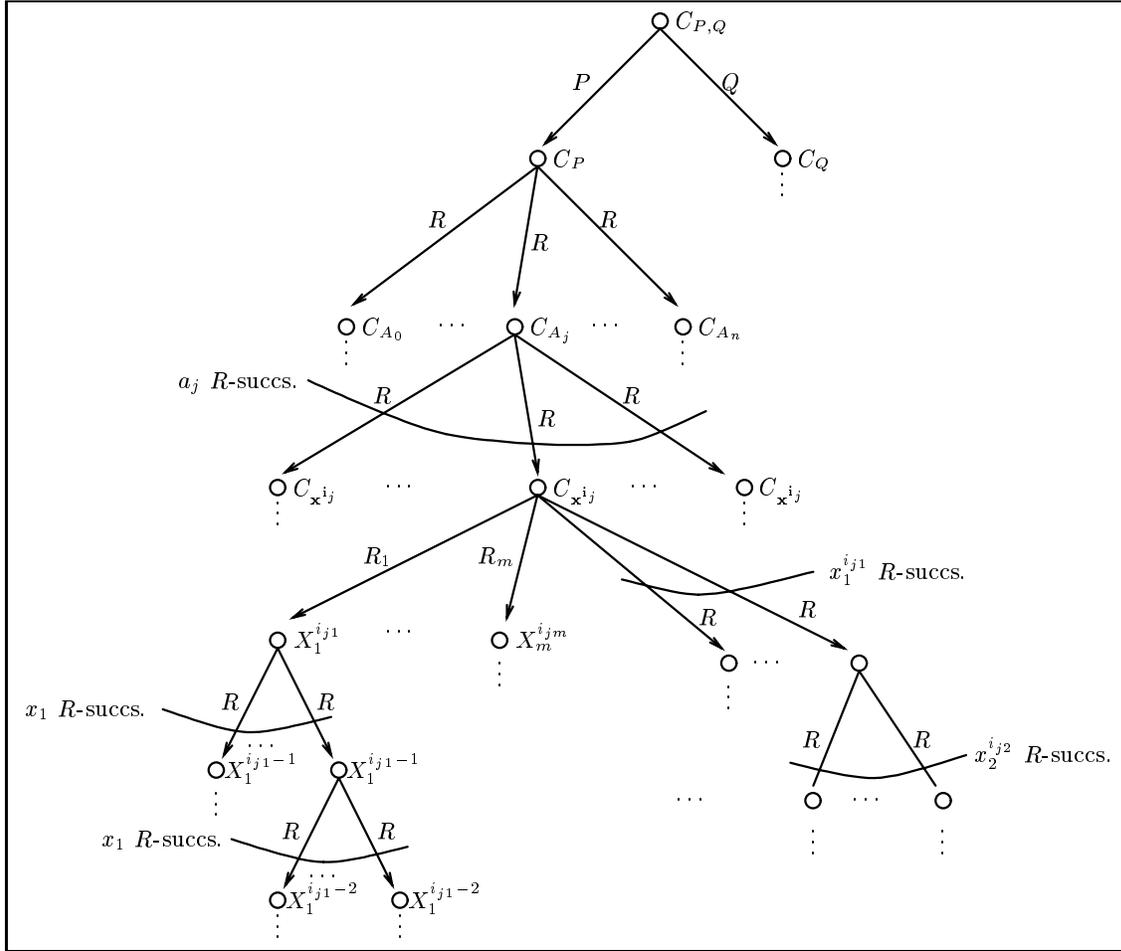


Figure 2: The intuitive structure of  $C_{P,Q}$

(b) This problem is solved by introducing features  $\mathbf{x}_i$  for each variable  $x_i$  and by making strong use of the concepts  $E_{\mathbf{x}_i}^R$  as defined above and  $\text{Inv}$ :

$$\text{Inv} := \prod_{1 \leq i \leq m} (\forall R. \top(\mathbf{x}_i) \sqcap P_=(\min(R \circ \mathbf{x}_i), \max(R \circ \mathbf{x}_i)) \sqcap P_=(\mathbf{x}_i, \max(R \circ \mathbf{x}_i))).$$

Using  $E_{\mathbf{x}_i}^R$ , we make sure that  $\mathbf{x}_i$ -successors are non-negative integers. The concept  $\text{Inv}$  is defined in such a way that  $R$ -successors of an instance  $a$  of  $\text{Inv}$  have the same  $\mathbf{x}_i$ -successor, which coincides with the  $\mathbf{x}_i$ -successor of  $a$ .

Using this concept at all levels of nested concepts, we can guarantee that all “relevant” individuals in a model of  $C_{P,Q}$  have the same  $\mathbf{x}_i$ -successor for each variable  $x_i$ .

(c) Addition can be realized by the aggregation function  $\text{sum}$ , and multiplication (and hence exponentiation) can be reduced to addition; for details see the explanation of the reduction concepts below.

Let  $\mathcal{D}$  be as described in Theorem 7. Then we can define the following abbreviations:

$$E_1^R := (\forall R.(P_{=1}(f))) \sqcap P_{=1}(\text{sum}(R \circ f)) \quad (\text{exactly 1 } R\text{-successor})$$

$$E_g^R := (\forall R.(P_{=1}(f))) \sqcap P_{=}(\text{sum}(R \circ f), g) \quad (\text{exactly } g^{\mathcal{I}}(x) \text{ } R\text{-successors})$$

$$E_n^R := \forall R. \left( \bigsqcup_{1 \leq i \leq n} (P_{=1}(f_i) \sqcap \prod_{j \neq i} \text{no}_{f_j}) \right) \sqcap \prod_{1 \leq i \leq n} P_{=1}(\text{sum}(R \circ f_i)) \quad (\text{exactly } n \text{ } R\text{-successors})$$

where  $\text{no}_{f_j}$  is the abbreviation for  $\forall f_j.(A \sqcap \neg A)$  mentioned in Section 2.

It is easy to see that each instance of  $E_1^R$  has exactly 1  $R$ -successor, and the concept  $E_g^R$  has already been explained above. Now, for an instance  $a$  of  $E_n^R$ , every  $R$ -successor has exactly one  $f_i$ -successor for some  $i$ ,  $1 \leq i \leq n$ , and this  $f_i$ -successor has value 1 (first line). The constraint on the concrete feature  $\text{sum}(R \circ f_i)$  (second line) makes sure that there is exactly one  $R$ -successor with an  $f_i$ -successor for each  $i$ , which implies that  $a$  has exactly  $n$   $R$ -successors.

More precisely, the reduction concept is built as follows and given in the Figures 3 and 4.

1. First, we define  $C_{P,Q}$  such that, for each interpretation  $\mathcal{I}$ , each instance  $x \in C_{P,Q}^{\mathcal{I}}$  has exactly one  $P$ -successor  $p$  in  $C_P^{\mathcal{I}}$  and exactly one  $Q$ -successor  $q$  in  $C_Q^{\mathcal{I}}$ . The individual  $p$  represents the polynomial  $P$ , and  $q$  represents  $Q$ ; see Concept 2. Concept 3 is similar to  $\text{lnv}$  and makes sure that for each  $j$ , the  $\mathbf{x}_j$ -successor of  $p$  is in  $\text{dom}(\mathcal{D})$  and the same as the  $\mathbf{x}_j$ -successor of  $q$ . Finally, Concept 4 makes sure that the value of the polynomial  $P$  when evaluated with the  $\mathbf{x}_j$ -successors (which are already ensured to be the same for  $p$  and for  $q$ ) is the same as of  $Q$ .
2. An instance  $p$  of  $C_P$  has
  - for each monomial  $A_j = a_j \mathbf{x}^{i_j}$  of  $P$  one  $R$ -successor which is an instance of  $C_{A_j}$  and represents the monomial  $A_j$ ; see Concept 5.
  - an  $s$ -successor which is the sum of the  $s$ -successors of its  $R$ -successors; see Concept 6.

Given that the  $s$ -successor of each  $R$ -successor of  $p$  is the value of the monomial  $A_j$ , the  $s$ -successor of  $p$  is the corresponding value of  $P$ , namely the sum over all monomials. Again, the concept  $\text{lnv}$  makes sure that each  $\mathbf{x}_i$ -successor of  $p$  coincides with the  $\mathbf{x}_i$ -successors of its  $R$ -successors.

3. For the monomials  $A_j$ , we use  $n + 1$  concepts  $C_{A_j}$ . The purpose of the last conjunct of Concept 7 is to achieve disjointness of these concepts  $C_{A_j}$ . An instance  $a$  of  $C_{A_j}$  has  $a_j$   $R$ -successors, each of them representing  $\mathbf{x}^{i_j}$ ; see Concept 9. The last conjunct makes sure that the  $s$ -successor (representing the value of  $A_j$ ) is computed correctly: Since  $a$  has  $a_j$   $R$ -successors, each of them representing  $\mathbf{x}^{i_j}$ , the  $s$ -successor of  $a$  is the sum over the  $s$ -successors of its  $R$ -successors.
4.  $C_{\mathbf{x}^{i_j}}$  is more complicated. An instance  $c$  of it has two different kinds of role successors:

$$C_{P,Q} := E_1^P \sqcap E_1^Q \sqcap \forall P.C_P \sqcap \forall Q.C_Q \sqcap \quad (2)$$

$$\sqcap_{1 \leq j \leq m} \left( P_{=}(\text{sum}(P \circ \mathbf{x}_j), \text{sum}(Q \circ \mathbf{x}_j)) \right) \sqcap \quad (3)$$

$$P_{=}(\text{sum}(P \circ s), \text{sum}(Q \circ s)) \quad (4)$$

$$C_P := E_{n+1}^R \sqcap \sqcap_{0 \leq i \leq n} (\exists R.C_{A_i}) \sqcap \quad (5)$$

$$\text{lnv} \sqcap P_{=}(s, \text{sum}(R \circ s)) \quad (6)$$

$$C_{A_j} := E_{a_j}^R \sqcap \forall R.C_{\mathbf{x}^{i_j}} \sqcap E_j^H \sqcap \quad (7)$$

$$\text{lnv} \sqcap P_{=}(s, \text{sum}(R \circ s)) \quad (8)$$

$$C_{\mathbf{x}^{i_j}} := \text{Exp}_{\mathbf{x}^{i_j}} \sqcap \text{Mult}_1^m \quad (9)$$

Figure 3: The reduction concept  $C_{P,Q}$  and some of its subconcepts.

- For each of the  $m$  factors  $x_k^{i_{jk}}$  in  $\mathbf{x}^{i_j}$ ,  $c$  has one  $R_k$ -successor in  $X_k^{i_{jk}}$ , whose  $s_k$ -successor stands for its value  $x_k^{i_{jk}}$ . The concept  $\text{Exp}_{\mathbf{x}^{i_j}}$  implies this fact. In  $\text{Exp}_{\mathbf{x}^{i_j}}$ , we use the second conjunct instead of  $\text{lnv}$  to propagate the value of  $x_k$  down to the according subtree. The last conjunct of  $\text{Exp}_{\mathbf{x}^{i_j}}$  makes sure that the respective values  $s_k$  are propagated upwards to  $c$ .
- Then, in order to multiply the  $m$  factors  $x_k^{i_{jk}}$ , we make use of the concept  $\text{Mult}_1^m$  explained below.

Again, the  $s$ -successor of  $c$  denotes the value of this calculation, namely  $\mathbf{x}^{i_j}$ .

5. For  $X_k^i$ , we have to distinguish two cases : If  $i = 0$ , then the value associated to this factor is 1; see the concept  $X_k^0$ . Otherwise, an instance  $y$  of  $X_k^i$  is the root of an  $x_k$ -ary  $R$ -tree of depth  $i$  where the  $s$ -successor of each node is the sum of the  $s$ -successors of its  $R$ -successors. Finally, the  $s$ -successor of a node one level above the leaves (which represents  $x_k^1$ ) equals its  $\mathbf{x}_k$ -successor—which is the same for all nodes in the whole tree. Since  $\text{dom}(\mathcal{D})$  is only required to contain the non-negative integers, we have to ensure that all  $\mathbf{x}_k$ -successors are non-negative integers. This is realized by making use of the concept  $E_{x_k}^R$ . Thus, we use the possibilities to construct trees and to sum up in order to compute exponentiation.
6. Finally, the situation in which we start multiplication looks as follows: An instance  $u$  of  $\text{Mult}_1^m$  is at the root of the multiplication tree,  $u$  is also an instance of  $C_{\mathbf{x}^{i_j}}$ , and we want to multiply all  $m$   $s_k$ -successors of  $u$ . To this purpose, we attach an  $R$ -tree

$$\text{Exp}_{\mathbf{x}^i j} := \prod_{1 \leq k \leq m} \left( E_1^{R_k} \sqcap P_{=}(\mathbf{x}_k, \text{sum}(R_k \circ \mathbf{x}_k)) \sqcap \right. \quad (10)$$

$$\left. \forall R_k. X_k^{i j k} \sqcap P_{=}(\mathbf{s}_k, \text{sum}(R_k \circ \mathbf{s}_k)) \right) \quad (11)$$

$$\text{Mult}_m^m := P_{=}(s, s_m) \quad (12)$$

$$\text{Mult}_k^m := E_{s_k}^R \sqcap P_{=}(s, \text{sum}(R \circ s)) \sqcap \forall R. \text{Mult}_{k+1}^m \sqcap \quad (13)$$

$$\prod_{\ell=k+1}^m \left( P_{=}(s, \text{sum}(R \circ s)) \sqcap P_{=}(s, \text{sum}(R \circ s)) \right) \quad (14)$$

$$X_k^0 := P_{=1}(s) \sqcap E_{\mathbf{x}_k}^R \quad (15)$$

$$X_k^1 := E_{\mathbf{x}_k}^R \sqcap P_{=}(s, \mathbf{x}_k) \quad (16)$$

$$X_k^\ell := E_{\mathbf{x}_k}^R \sqcap \forall R. X_k^{\ell-1} \sqcap P_{=}(s, \text{sum}(R \circ s)) \sqcap \quad (17)$$

$$P_{=}(s, \text{sum}(R \circ s)) \sqcap P_{=}(s, \text{sum}(R \circ s)) \sqcap P_{=}(s, \text{sum}(R \circ s)), \ell \geq 2$$

Figure 4: Subconcepts of  $C_{P,Q}$  used for the representation of calculations.

of depth  $m - 1$  to  $u$ . This tree is, at level  $k$ , of outdegree  $s_k$ . At level  $m - 1$ , we make sure that the  $s_m$ -successors coincide with the  $s$ -successor. Again, we sum up the values from the bottom to the top by using the concept  $P_{=}(s, \text{sum}(R \circ s))$ , and we make sure that all nodes have the same  $s_i$  successor by a concept similar to  $\text{Inv}$ ; see Concept 14.

It remains to be shown that  $C_{P,Q}$  is satisfiable iff  $P(\mathbf{x}) = Q(\mathbf{x})$  admits a solution in the non-negative integers.

“ $\Leftarrow$ ” The construction of a model  $M$  for  $C_{P,Q}$  from  $P, Q$ , and a solution  $n_1, \dots, n_m \in \mathbb{N}^m$  for  $\mathbf{x}$  is not difficult.  $M$  can be constructed along the explanations given for  $C_{P,Q}$  in the following way: We start at the bottom of the tree  $M$  by introducing instances  $x_k^1$  of

- $X_k^1$  that have  $n_k$   $R$ -successors, each of them having 1 as  $f$ -successor (due to the use of  $E_{\mathbf{x}_k}^R$ ),  $n_k$  as  $\mathbf{x}_k$  successor, and  $n_k$  as  $s$ -successor, and instances  $x_k^0$  of
- $X_k^0$  that have  $n_k$   $R$ -successors, each of them having 1 as  $f$ -successor,  $n_k$  as  $\mathbf{x}_k$  successor, and 1 as  $s$ -successor.

Then, for each monomial  $\mathbf{x}^{i_j}$ , the corresponding subtrees representing  $n_k^{i_{jk}}$  are built: Starting with (copies of)  $x_k^1$  and  $x_k^0$ , we build trees of depth  $i_{jk}$  and degree  $n_k$ . Next, instances  $c$  of  $C_{\mathbf{x}^{i_j}}$  are introduced, where each  $c$  has as  $R_k$ -successor the subtree representing the factor  $n_k^{i_{jk}}$  in  $n_1^{i_{j1}} \dots n_m^{i_{jm}}$ . Now, we have to append another subtree to each  $c$ , namely the one representing the multiplication of the values  $n_k^{i_{jk}}$ . This tree is of depth  $m - 1$  and degree  $n_k^{i_{jk}}$  at level  $k$ . The remaining construction is straightforward: We first take  $a_j$  disjoint copies of the  $c$ 's standing for  $C_{\mathbf{x}^{i_j}}$  (including the corresponding subtree) as  $R$ -successors of an instance  $a$  of  $C_{A_j}$ , then we append these  $a$ 's as  $R$ -successors of an instance of  $p$  of  $C_P$ . We suppose that the same construction has been carried out for  $Q$ , which lead to an instance  $q$  of  $C_Q$ . Finally,  $p$  and  $q$  are  $P$  (resp.  $Q$ ) -successors of an instance  $c$  of  $C_{P,Q}$ .

All over the tree constructed in this way, the  $s$ -successor of an individual equals the sum over the  $s$ -successors of its  $R$ -successors, and all individuals have the same  $\mathbf{x}_k$ -successor. The fact that a solution  $n_1, \dots, n_m \in \mathbb{N}^m$  for  $\mathbf{x}$  has to be used for this construction is reflected in the fact that, due to the definition of  $C_{P,Q}$ ,  $p$ 's  $s$ -successor has to coincide with  $q$ 's  $s$ -successor.

“ $\Rightarrow$ ” Given a model  $M$  for  $C_{P,Q}$  with  $c \in C_{P,Q}^I$ , due to the presence of  $\text{lnv}$  and similar concepts in  $C_{P,Q}$ , all  $\mathbf{x}_i$ -successors of all “relevant” role successors of  $c$  coincide—where “relevant” role successors are those whose existence is explicitly required by  $C_{P,Q}$ . Again, following the description of  $C_{P,Q}$ , it is easy to see that  $(\mathbf{x}_1^I(c), \dots, \mathbf{x}_m^I(c))$  is a solution for  $P(\mathbf{x}) = Q(\mathbf{x})$ . Due to the use of the concepts  $E_{x_i}^R$ , this solution is in  $\mathbb{N}^m$ .

Hence satisfiability and thus subsumption of  $\mathcal{ALC}(\mathcal{D})$ -concepts is undecidable for concrete domains  $\mathcal{D}$  as described in Theorem 7. ■

We want to emphasize that  $C_{P,Q}$  does not make any use of the possibility to apply aggregation functions to feature chains, i.e., wherever a subconcept of  $C_{P,Q}$  contains  $\Sigma(R \circ f)$  for some aggregation function  $\Sigma$ ,  $f$  is a feature name (and not a complex feature chain or concrete feature).

### 3.2 Tightening the result

A closer investigation of the concept  $C_{P,Q}$  reveals that (a) negation occurs only in the concept  $\text{no}_f$ , (b) the only place where existential restriction occurs is in the concepts  $C_P$  and  $C_Q$ , and (c) the only place where disjunction  $\sqcup$  occurs is in the concepts  $E_n^R$  describing individuals having exactly  $n$   $R$ -successors.

It can be shown that the concepts  $\text{no}_f$ ,  $E_n^R$  and  $C_P$  can be rewritten into concepts without negation, disjunction and existential restriction, by extending only slightly the set of concrete predicates. Hence, the reduction concept  $C_{P,Q}$  can be written using only conjunction  $\sqcap$  and universal value restriction  $\forall R.C$ . As introduced in [Baader,1996], let  $\mathcal{FL}_0$  denote the set of those concepts that are built using conjunction and universal value restriction

only, and let  $\mathcal{FL}_0(\mathcal{D}+\Sigma)$  denote the extension of this language by concrete domains with aggregation. Then the following undecidability result is an immediate consequence of the possibility to rewrite the reduction concept  $C_{P,Q}$  without using negation, disjunction, and existential restriction.

**Theorem 8** For a concrete domain  $\mathcal{D}$  where

- $\text{dom}(\mathcal{D})$  includes the non-negative integers  $\mathbb{N}$ ,
- $\text{pred}(\mathcal{D})$  contains, for all non-negative integers  $n$ , (unary) predicates  $P_{=n}$  that test for equality with  $n$ , the (binary) equality predicate  $P_{=}$ , and the (binary) inequality predicate  $P_{\neq}$ ,
- $\text{agg}(\mathcal{D})$  contains  $\text{min}$ ,  $\text{max}$ ,  $\text{sum}$ ,

satisfiability and subsumption of  $\mathcal{FL}_0(\mathcal{D}+\Sigma)$ -concepts is undecidable.

**Remarks:** (a) Admissible concrete domains as defined in [Baader & Hanschke,1991] are closed under negation, hence the presence of a predicate  $P_{=}$  in  $\text{pred}(\mathcal{D})$  implies the presence of its negation  $P_{\neq}$ . Thus, for admissible domains, only the unary predicates  $P_{=n}$  are required in addition to the preconditions of Theorem 7.

(b) We recall that according to the semantics of  $\mathcal{FL}_0(\mathcal{D}+\Sigma)$ , an individual  $x$  can only be an instance of the concept  $P_{\neq}(f, g)$  if  $x$  has an  $f$ - as well as a  $g$ -successor in the concrete domain  $\text{dom}(\mathcal{D})$ .

**Proof:** It remains to define  $\mathcal{FL}_0(\mathcal{D}+\Sigma)$ -concepts  $\text{no}'_f$ ,  $E'_n{}^R$  and  $C'_P$  which can play the rôle of  $\text{no}_f$ ,  $E_n^R$  and  $C_P$  in the reduction concept  $C_{P,Q}$  of the proof of Theorem 7.

$\text{no}'_f$  : This concept is used to make sure that an individual has no  $f$ -successor. It can be clearly replaced by

$$\text{no}'_f := \forall f. P_{\neq}(g, g),$$

where  $P_{\neq}(g, g)$  plays the rôle of the empty concept  $A \sqcap \neg A$  used in the definition of  $\text{no}_f$ .

$E'_n{}^R$  : Given a concrete domain  $\mathcal{D}$  that provides, for all non-negative integers  $n$ , a unary predicate  $P_{=n}$  that tests for equality with  $n$ , we can define a concept  $E'_n{}^R$  whose instances have exactly  $n$   $R$ -successors:

$$E'_n{}^R := \forall R. P_{=1}(f) \sqcap P_{=n}(\text{sum}(R \circ f)).$$

Obviously, replacing  $E_n^R$  by  $E'_n{}^R$  in  $C_{P,Q}$  preserves its property of serving as a reduction concept for Hilbert's 10th problem. Avoiding existential restriction in  $C_P$  is more complicated.

$C'_P$  : In  $C_P$ , existential restriction is used to make sure that for each monomial  $A_j$  there is one  $R$ -successor representing this monomial (the uniqueness of this  $R$ -successor stems from the fact that there are exactly  $n + 1$   $R$ -successors and that the  $C_{A_j}$  are mutually disjoint). This can also be expressed by introducing for each  $j$  exactly one  $R_j$ -successor (using  $E_1^{R_j}$ ), and then using universal value restrictions to make sure that the  $R_j$ -successor is an instance of  $C_{A_j}$ . Additionally, the  $\mathbf{x}_j$ -successors have to be propagated to the  $R_j$ -successors. All this is ensured by the first line of  $C'_P$ .

$$C'_P := \bigcap_{0 \leq j \leq n} \left( E_1^{R_j} \sqcap \forall R_j. C_{A_j} \sqcap \bigcap_{0 \leq \ell \leq m} P_=(\mathbf{x}_\ell, \text{sum}(R_j \circ \mathbf{x}_\ell)) \sqcap \right. \\ \left. P_=(s_j, \text{sum}(R_j \circ s)) \right) \sqcap \\ \text{Add}_{s_0, \dots, s_n}$$

It remains to enforce that the sum of all  $s$ -successors of all  $R_j$ -successors of an instance  $p$  of  $C'_P$  coincides with  $p$ 's  $s$ -successor. For this purpose, we make sure that  $p$  has an  $s_j$ -successor which coincides with the  $s$ -successor of its  $R_j$ -successor. Then the concept  $\text{Add}_{s_0, \dots, s_n}$  is used to sum up  $p$ 's  $s_j$ -successors. It is defined inductively as follows:

$$\text{add}_{s_0, s_1}^t := \forall R_{01}. (E_{g_0}^{G_0} \sqcap E_{g_1}^{G_1} \sqcap P_{\neq}(g_0, g_1)) \sqcap \\ P_{=1}(\text{max}(R_{01} \circ g_0)) \sqcap P_{=0}(\text{min}(R_{01} \circ g_0)) \sqcap \\ P_{=1}(\text{max}(R_{01} \circ g_1)) \sqcap P_{=0}(\text{min}(R_{01} \circ g_1)) \sqcap \\ P_=(s_0, \text{sum}(R_{01} \circ g_0)) \sqcap P_=(s_1, \text{sum}(R_{01} \circ g_1)) \sqcap \\ E_t^{R_{01}}$$

The idea underlying this addition is the following. First, the addition of  $n + 1$  numbers is reduced to the addition of two numbers: Therefore, the  $s_0$ - and the  $s_1$ -successor of  $p$  are summed up and the result is stored as  $s_{01}$ -successor of  $p$ . Next, the  $s_{01}$ - and the  $s_2$ -successor are summed up and the result is stored as  $s_{012}$ -successor of  $p$ , and so forth, until only two arguments are left. The sum of these last numbers is the result of the whole addition, and therefore stored as  $s$ -successor of  $p$ .

The addition of two numbers (given as  $s_0$ - and  $s_1$ -successors) and the storage of the result as  $t$ -successor is realized by the concept  $\text{add}_{s_0, s_1}^t$  given above: Let  $p$  be an instance of  $\text{add}_{s_0, s_1}^s$ , let  $s_0$  be  $p$ 's  $s_0$ -successor, and let  $s_1$  be  $p$ 's  $s_1$ -successor. From the construction of  $C_{P,Q}$  it follows that  $s_0, s_1$  are non-negative integers. We make sure that the number of  $R_{01}$ -successors of  $p$  equals  $s_0 + s_1$ . Additionally,  $p$ 's  $t$ -successor equals the number of its  $R_{01}$ -successors, which is  $s_0 + s_1$ .

To enforce that the number of  $R_{01}$ -successors of  $p$  equals  $s_0 + s_1$ , we need all but the last line of concept  $\text{add}_{s_0, s_1}^s$ . The idea is to partition the  $R_{01}$ -successors into those contributing to  $s_0$  and those contributing to  $s_1$ .  $R_{01}$ -successors contributing to  $s_0$  have 1 as  $g_0$ -successor and 0 as  $g_1$ -successor, whereas  $R_{01}$ -successors contributing to  $s_1$  have 0 as  $g_0$ -successor and 1 as  $g_1$ -successor. The first line of  $\text{add}_{s_0, s_1}^s$  ensures that all  $g_i$ -successors of all  $R_{01}$ -successors of  $p$  are non-negative integers (by using auxiliary roles  $G_0, G_1$ ) and

that the  $g_0$ -successor always differs from the  $g_1$ -successor. The next two lines make sure that  $g_i$ -successors of  $R_{01}$ -successors are between 0 and 1. Together with the fact that they are non-negative integers and different, we have that each  $R_{01}$ -successor has either 1 as  $g_0$ -successor and 0 as  $g_1$ -successor or vice versa. The fourth line states that the number of  $R_{01}$ -successors representing  $s_0$  (i.e., the ones having 1 as  $g_0$ -successor) is  $s_0$ , and that the number of  $R_{01}$ -successors representing  $s_1$  is  $s_1$ . Finally, the last line enforces that the number of  $p$ 's  $R_{01}$ -successors coincides with its  $s$ -successor.

Again, replacing  $C_P$  by  $C'_P$  (respectively  $C_Q$  by  $C'_Q$ ) in  $C_{P,Q}$  preserves its property of serving as a reduction concept for Hilbert's 10th problem, which is—in contrast to the initial one—an  $\mathcal{FL}_0(\mathcal{D}+\Sigma)$ -concept.

Undecidability of subsumption follows from undecidability of satisfiability because a concept  $C$  is satisfiable iff it is not subsumed by an unsatisfiable concept, and because the  $\mathcal{FL}_0(\mathcal{D}+\Sigma)$ -concept  $P_{\neq}(f, f)$  is such an unsatisfiable concept. ■

## 4 Conclusion

Reasoning with constraints involving aggregation functions is a crucial task for many advanced information systems like decision support and on-line-analytical processing systems, data warehouses, and (statistical) databases [Ross *et al.*,1998; Gupta *et al.*,1995; Mumick & Shmueli,1995; De Giacomo & Naggar,1996; Levy & Mumick,1996; Srivastava *et al.*,1996]. The more the amount of data that are processed by these systems grows, the more important become aggregation functions for summarizing, consolidating and analyzing these large amounts of data. Hence, traditional techniques for query rewriting, query optimization, view maintenance, etc. must be extended such that they are able to cope with aggregation functions.

The two undecidability results presented in this paper indicate that this task will be difficult. The aggregation functions `min`, `max`, `sum` that suffice to obtain undecidability are the most “well-behaved” ones: aggregation functions like `count` or `average` are much more difficult to handle. For example, `min`, `max`, `sum` are monotonic, i.e., if  $S \subseteq S'$ , then

$$\begin{aligned} \min(S) &\geq \min(S'), \\ \max(S) &\leq \max(S'), \\ \text{sum}(S) &\leq \text{sum}(S'), \end{aligned}$$

whereas these relations cannot be established for `count` or `average`. Furthermore, they are “compositional” in the sense that the aggregation  $f \in \{\min, \max, \text{sum}\}$  of two disjoint multisets  $S, S'$  can be computed using  $f, f(S), f(S')$  only—which does not hold, for example, for `average`. Hence, our undecidability result cannot be said to be caused by using a too powerful set of aggregation functions.

Arguing from another perspective,  $\mathcal{ALC}(\mathcal{D} + \Sigma)$  is a rather expressive Description Logic and it might not be very surprising that adding aggregation to  $\mathcal{ALC}(\mathcal{D})$  leads to undecidability. In contrast,  $\mathcal{FL}_0$  is, to our knowledge, the weakest Description Logic ever

considered. It is of such a low expressive power that subsumption between two  $\mathcal{FL}_0$ -concepts can be reduced to answering conjunctive queries: given two  $\mathcal{FL}_0$ -concepts  $C_1$  and  $C_2$ ,  $C_1$  subsumes  $C_2$  if and only if an individual  $x$  of an extensional database  $\text{edb}_{C_1(x)}$  constructed from  $C_1$  is in the answer set of a conjunctive query  $q_{C_2}$  constructed from  $C_2$ . This reduction is, for several reasons, not possible for  $\mathcal{FL}_0(\mathcal{D}+\Sigma)$ -concepts. However, it leads to the speculation that (intensional) reasoning for conjunctive queries with (simple) aggregation functions and built-in predicates is of high computational complexity.

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